# ISI B. Math.
# Physics I
# Mid Semetral Exam
# Total Marks: 100

Answer any five questions. All questions carry equal marks.

1. Consider the motion of a particle of mass $m$ under the influence of a force $\mathbf{F} = -k\mathbf{r}$ where $k$ is a positive constant and $\mathbf{r}$ is the position vector of the particle.
(a) Prove that the motion of the particle lies in a plane.
(b) Find the position of the particle as a function of time, assuming that at $t = 0, x = a, y = 0$ and $v_x = 0, v_y = v_0$.
(c) Show that the orbit is an ellipse.
(d) Find the period.
(e) Does the motion of the particle obey Kepler's laws of planetary motion?

2. Consider a pendulum of length $l$ and a bob of mass $m$ at its end moving through oil. The massive bob undergoes small oscillations, but the oil retards the bob's motion with a resistive force proportional to the speed with $F_{\text{res}} = -2m\sqrt{\frac{g}{l}}(l\dot{\theta})$. The bob iis initially pulled back at $t = 0$ with $\theta = \alpha$ and $\dot{\theta} = 0$.
(a) Find the angular displacement $\theta$ and the velocity $\dot{\theta}$ as a function of time.
(b) Show that at late times such that $\sqrt{\frac{g}{l}}t \gg 1$ the total mechanical energy of the pendulum decreases as $(\text{const}) \times t^2 e^{-2\sqrt{\frac{g}{l}}t}$
(c) Now assume that the pendulum is taken out of oil and put in a friction free medium amd started out with the same initial conditions. Make a sketch of the plot of $\theta$ vs $\dot{\theta}$ for a given energy fixed by the initial conditions. Such a plot is known as a phase space plot.

3. (a) Check whether the following force is conservative. If the answer is yes, find the potential energy for it.

$$\mathbf{F} = (6abz^3y - 20bx^3y^2)\mathbf{i} + (6abxz^3 - 10bx^4y)\mathbf{j} + 18abxz^2y\mathbf{k}$$

where $a, b$ are constants.
(b) Two particles with mass $m_1$ and $m_2$ interact with gravitational forces

($\mathrm{F} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$). They start out from rest a distance $\rho$ apart and are allowed to fall into each other. How long does it take for them to collide?
(You may need to use the following result:

$$\int_0^1 \frac{dx}{\sqrt{\frac{1}{x}-1}} = \frac{\pi}{2}.$$

(*Hint: Think in terms of conservation laws*)

4. (a) The equation of motion of a point electric charge $e$ of mass $m$ in the field of a magnetic monopole of strength $g$ at the origin is

$$m\ddot{\mathbf{r}} = -ge\frac{\dot{\mathbf{r}} \times \mathbf{r}}{r^3}$$

Show that the kinetic energy $T = \frac{1}{2}m\dot{\mathbf{r}}^2$ is a constant of the motion.
(b) The orbit of a particle moving under a central force is $r\theta =$ constant. Determine the potential energy as a function of $r$.

5. A particle of mass $\sqrt{2}m$ collides elastically with a target particle of mass $m$ at rest.
(a) Show that the maximum angle through which the incident particle can be scattered is $45^\circ$.
(b) Sketch what the same collision would look like in the centre of mass co-ordinate system.

6. Consider the one dimensional potential

$$U(x) = -Wd^2\frac{(x^2+d^2)}{(x^4+8d^4)}$$

(a) Where are the equilibrium points?
(b) Are they stable or unstable?
(c) Sketch the potential.
(d) Is the motion bounded or unbounded ?
(e) Find the turning points for $E = -\frac{W}{8}$. The value of $W$ is a positive constant and so is $d$ .
(Hint: Define $Z(y) = \frac{U(x)}{W}$ where $y = \frac{x}{d}$ and sketch $Z(y)$ vs $y$ to sketch the potential. Use parts (a) and (b) as guides to sketching the potential.)